Chaotic noisy transport of electron pairs in a superconducting junction device: Thermal-inertia ratchets

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Chaotic noisy transport of electron pairs in a superconducting junction device (thermal-inertia ratchets) is investigated. The study shows that when the temperature is low enough, the transport of the electron pairs can be mainly chaotic; when the temperature is high enough, it can be mainly stochastic. By controlling the temperature and the amplitude of the input ac signal, the current of electron pairs can be reversed.

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Recently, there has been an increasing interest in studying the net voltage [1,2] (i.e., a nonzero dc voltage with a zero dc current) and the dc current-voltage characteristics [3,4] in Josephson junctions (superconducting junctions) with noise. It is reported that asymmetric noise can produce a net voltage [1,2], and dc voltage rectification [3,4]. We have shown that correlated symmetric noise can also produce a net voltage [2,5], which stems from a symmetry breaking of the system induced by the correlation between additive and multiplicative noises. In Ref. [6], Zapata *et al.* investigated the dc current-voltage characteristics for an asymmetric dc device with three Josephson junctions (depicted in Fig. 1) threaded by a magnetic flux and driven by a periodic signal and additive noise. But they did not consider the case in the presence of additive and multiplicative noise, especially the case when the additive and multiplicative noises are correlated. In Ref. [7], we studied the net voltage, the dc current-voltage characteristics, and the mean first passage time for this device in the case of environmental perturbation, together with thermal fluctuation. (The environmental perturbation can be described by multiplicative noise in the Langevin equation [8–10], and the thermal fluctuation by additive Gaussian white noise [11].)

However, all of the above work was focused on Josephson junctions in the overdamped limit. In this paper, we will study the flux of the electron pairs of the device [6,7] in the underdamped case, and try to get the properties of the transport of the electron pairs. We focus on this device (see Fig. 1) formed by Josephson junctions whose phase is a classical variable and which can be adequately described by the "resistively shunted junction" model [12,13]. Thus, the phase ϕ_i across Josephson junction i on the left arm obeys the equation $(i=1,2)$

$$I_l(t) = J_i \sin(\phi_i) + \frac{\hbar}{2eR_i}\dot{\phi}_i + \frac{\hbar C_i}{2e}\ddot{\phi}_i, \tag{1}$$

where $I_l(t)$ is the current through the left arm, and R_i, C_i, and J_i are the resistance, capacitance, and critical current of junction i. For simplicity, we assume here that the two junctions in series are identical. We take $C_1=C_2=2C_l$, $R_1=R_2=R_l/2$, and $J_1=J_2=J_l$. The total voltage drop across the two junctions is $V=V_1+V_2$, where $V_i=(\hbar/2e)\dot{\phi}_i$. If $\phi_1(t)$ is a solution for the first junction, then $\phi_2(t)=\phi_1(t)=\phi_l(t)/2$ is also a solution for the second junction [14]. This implies $V=\dot{\phi}_l\hbar/2e$, with ϕ_l satisfying the equation

$$I_l(t) = J_l \sin(\phi_l/2) + \frac{\hbar}{2eR_l}\dot{\phi}_l + \frac{\hbar C_l}{2e}\ddot{\phi}_l. \tag{2}$$

Hence, a series of two identical Josephson junctions can be described by the same equation as a single junction, with the only difference that in the sine function the argument $\phi/2$ $(\phi=\phi_l)$ occurs [15]. On the right arm, the phase across the single junction obeys an equation that reads as in Eq. (1) with the labels l and i replaced by r. The total current through the device is $I(t)=I_l(t)+I_r(t)$, which is marked in Fig. 1. Here, we assume that the loop inductance L is very small [i.e., $LI(t) \ll \Phi_e$; Φ_e is the external magnetic flux] and the length of the loop of the device is much larger than the penetrating depth of the electron pairs. Then, the phase around the loop yields $\phi_l-\phi_r=-\phi_e+2\pi n$, with $\phi_e=2\pi\Phi_1/\Phi_0$ $(\Phi_0=\frac{\hbar}{2e})$. So the phase satisfies the equation

$$\frac{\hbar C}{e}\ddot{\phi} + \frac{\hbar}{eR}\dot{\phi} = -J_l \sin\frac{\phi}{2} - J_r \sin(\phi+\phi_e) + I(t), \tag{3}$$

in which $\phi=\phi_l$, $R_r=R_l=R$, and $C_r=C_l=C$.

After considering the thermal fluctuation, Eq. (3) becomes

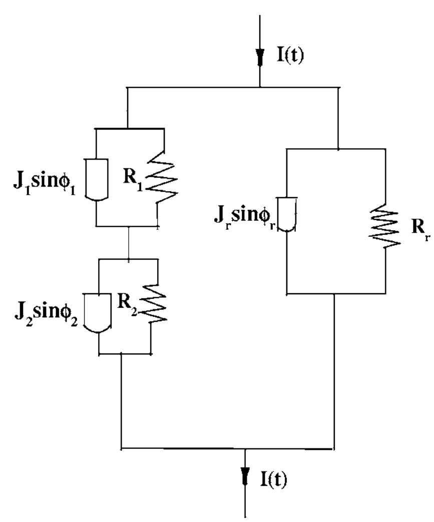

FIG. 1. The three-Josephson-junction device.

$$\frac{\hbar C}{e}\ddot{\phi} + \frac{\hbar}{eR}\dot{\phi} = -J_l \sin\frac{\phi}{2} - J_r \sin(\phi + \phi_e) + I(t) + \xi(t), \tag{4}$$

where $\xi(t)$ is the thermal Gaussian white noise with zero mean and correlation $\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t')$, with D proportional to the temperature T.

Next we feed the circuit with an ac current $I(t) = a\cos\omega t$. Then after assuming $\phi = \pi + 4\pi(x - x_0)$, $\phi_e = \pi/2$, and $J_r = J_l/2$, Eq. (4) reads

$$m\ddot{x} + \gamma\dot{x} = f(x) + a\cos\omega t + \xi(t), \tag{5}$$

in which $f(x) = -J_l[\cos 2\pi(x-x_0) + (1/2)\cos 4\pi(x-x_0)]$, $m = 4\pi\hbar C/e$, and $\gamma = 4\pi\hbar/(eR)$. The potential $U(x)$ in Eq. (5) $[\partial_x U(x) = -f(x)]$ is a ratchet, which has been the subject of extensive recent studies [16].

In the absence of noise ($D=0$), Eq. (5) has a variety of trajectories which can be obtained by numerical integration. Here what we are interested in is the chaotic trajectories produced by Eq. (5) in the absence of noise by controlling the parameters. For $D=0$, $\omega=0.67$, $m=1$, $\gamma=0.1$, $x_0=-0.19$, and $J_l=0.0995$ (we fix these parameters throughout this paper), and values of a in $0<a<10$, where the system exists on chaotic attractors, there is always a net drift of electron pairs to the right or to the left, i.e., electron pairs move on average either to the left or to the right. In the presence of noise ($D\neq 0$), the situation is different. Now the trajectories described by Eq. (5) begin to show a very complex behavior (chaotic noisy behavior) for the above fixed parameters of the system.

For the convenience of analysis and calculation, we write Eq. (5) as

$$\dot{x} = y, \quad \dot{y} = -\frac{\gamma y}{m} + g(x) + \frac{a}{m}\cos\omega t + \frac{1}{m}\xi(t),$$

where $g(x) = -\frac{1}{m}f(x)$. In the Stratonovich case, the Fokker-Planck equation for the probability density $P(x,y,t)$ corresponding to Eq. (6) is

$$\partial_t P = -y\partial_x P - \partial_y\left(-\frac{1}{m}y + g(x)\right)P + \frac{1}{m}(D + a\cos\omega t)\partial_y^2 P. \tag{6}$$

Equation (6) cannot be solved analytically even for the stationary case since detailed balance is broken and the probability flow is not zero, but it can be solved by applying numerical methods. In the following we carry out our numerical simulation directly using the Langevin equation (5). From Ref. [17] we can get the numerical algorithm

$$x(t+\Delta t) = x(t) + y\Delta t,$$

$$y(t+\Delta t) = y(t) + \left(-\frac{1}{m}y(t) + g(x(t)) + \frac{a}{m}\cos\omega t\right)\Delta t + x(t,\Delta t), \tag{7}$$

where $x(t,\Delta t) = r\sqrt{2D\Delta t/m}$, with a Gaussian random number r of zero mean and variance 1. Here we define the current J which is averaged over an ensemble of initial conditions for the average velocity. Therefore, the current has two different averages. The first average is over M initial conditions, which we take equally distributed in space (from $x=0$ to 4π), and with a zero initial velocity. For a fixed time t_j, we can obtain the first average $v_j = (\frac{1}{M})\sum_{i=1}^{M}\dot{x}_i(t_j)$. The second average is a time average. Since we take a discrete time for the numerical simulation, we have a discrete finite set of N different times t_j. Then the current is defined as $J = (1/N)\sum_{j=1}^{N} v_j$.

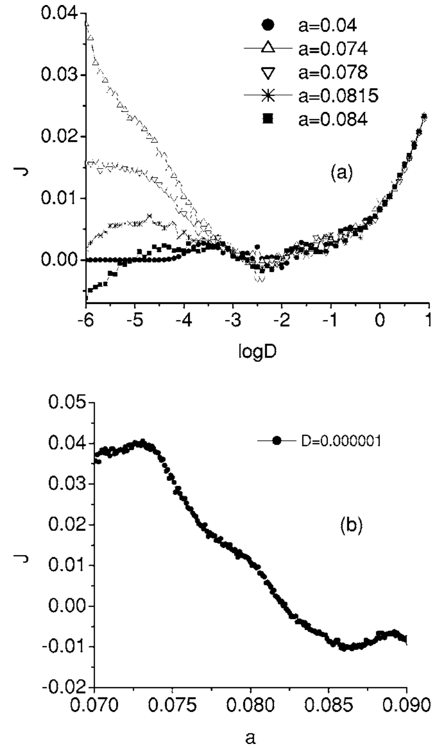

FIG. 2. (a) The current J versus the logarithm of the noise strength D for $a=0.04$, 0.074, 0.078, 0.0815, and 0.084, respectively, with the fixed parameters $\omega=0.67$, $\gamma=0.1$, $m=1$, $x_0=-0.19$, and $J_l=0.0995$. (b) The current versus the amplitude a of the ac input signal for $D=10^{-6}$ with $\omega=0.67$, $\gamma=0.1$, $m=1$, $x_0=-0.19$, and $J_l=0.0995$.

The numerical results are plotted in Figs. 2(a) and 2(b) for the current versus the noise strength and the current versus the amplitude of the ac input signal, respectively. Every point in the figures is calculated by taking the average of the $M=400$ initial conditions and the $N=10^5$ different discrete times (averaged by 4×10^7 points). Here the time step is taken as $\Delta t=0.01$. In order to guarantee that the system is in the stationary state, we take the time average after $t=1000$. This average is taken from $t=1000$ to 2000. The space from $x=0$ to 4π is divided into 400 ($x_i = 4\pi i/400$, $i=1,2,\ldots,400$). The initial conditions are $x_i(t=0)=0$ ($i=1,2,3,\ldots,400$). In Fig. 2(a), we plot the current versus the logarithm of the additive noise strength for $a=0.04$, 0.074, 0.078, 0.0815, and 0.084, respectively, with the fixed parameters $\omega=0.67$, $\gamma=0.1$, $m=1$, $x_0=-0.19$, and $J_l=0.0995$. The figure shows that when the noise strength is small enough, the properties of the transport of the electron pairs are controlled by chaos of the system; when the noise

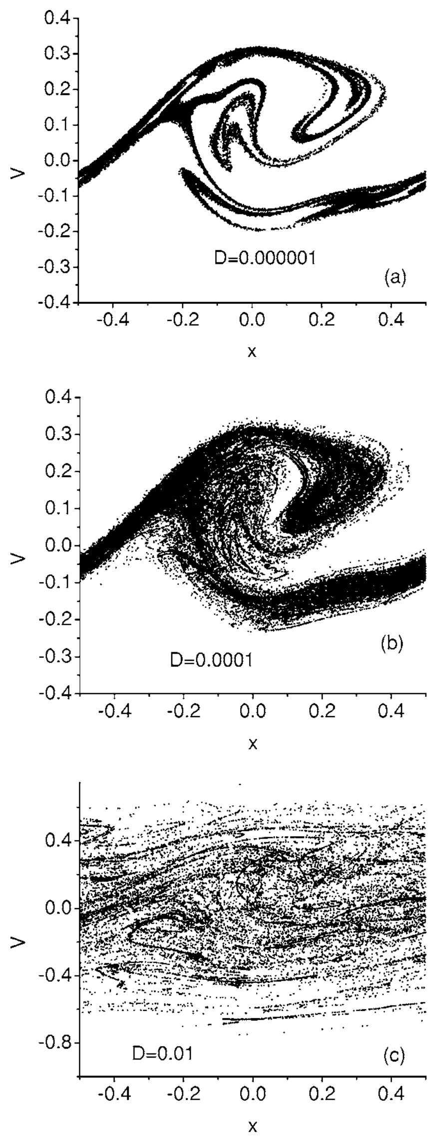

FIG. 3. The Poincaré map for different values of D [D=(a) 10^{-6}, (b) 10^{-4}, and (c) 10^{-2}] with $a=0.080\,92$ and the above fixed parameters $\omega=0.67$, $\gamma=0.1$, $m=1$, $x_0=-0.19$, and $J_l=0.0995$.

strength is large enough, the properties of the transport of the electron pairs are controlled by the noise. Because the thermal Gaussian white noise is proportional to the temperature T, we can say that, if the temperature is low enough, the transport of the electron pairs is mainly chaotic (controlled by the internal chaos of the system); if the temperature is large enough the transport of the electron pairs is mainly stochastic (controlled by the temperature). There is a critical value D_0 of D ($D_0 \approx 10^{-3.4}$). When $D < D_0$, by varying the values of the amplitude of the input oscillatory signal, we can get different values of the current; but, when $D > D_0$, the values of the current are almost unvaried for different values of a [except for the values of a in Fig. 2(a); we have also simulated a lot of values of a from 0 to 10 and the study shows that the system has the same behavior]. We call D_0 "the critical value" of the chaotic noisy transport of the electron pairs. By controlling the values of a and D (or the temperature), we can reverse the transport of the electron pairs. In Fig. 2(b), we present the current J versus the amplitude a of the input oscillatory signal for $D=10^{-6}$ with the above fixed parameters. This figure represents the appearance of current reversal of the electron pairs, by controlling the values of a. In this figure, we only show a limited range of values of a, which ranges from 0.07 and 0.09. The dependence of J versus a for a much broader range of a values should be displayed ($0 \leqslant a < \infty$). When a is less than 0.07, with decreasing values of a, the current decreases (when $a=0$, $J=0$); when a is larger than 0.09, with the increase of the values of a, the absolute current $|J|$ decreases (when $a \to \infty$, J tends to zero).

To illustrate clearly the properties of the chaotic noisy transport of the electron pairs, in Figs. 3(a)–3(c) we depict the Poincaré map for different values of D ($D=10^{-6}$, 10^{-4}, and 10^{-2}, respectively) with $a=0.080\,92$ and the above fixed parameters. In Fig. 3(a), we can observe chaotic attractors in the presence of noise; but with increasing noise strength, in Fig. 3(b) the chaotic attractors are destroyed by the noise; with further increase in noise strength, in Fig. 3(c) we cannot observe the chaotic attractors, since they are completely destroyed by the noise.

I have noted that the average current for the chaotic transport of the particles has been studied for a periodic asymmetric potential of the ratchet type in Ref. [18]. It has been shown that the average current could exhibit sign reversals [18]. Our study is different from the one in Ref. [18]. In Ref. [18] only the case in the presence of chaos was considered, while our study has considered the case under the presence of noise and chaos. In addition, in Refs. [19,20], Machura and co-workers have considered Brownian motor transport which satisfies Eq. (5) [only the potential ratchets are different for Eq. (5) in our paper and the model studied in Refs. [19,20]]. But they considered only the case when the thermal noise strength is larger than 10^{-2} in Ref. [19] and equals 10^{-3} or 5×10^{-3} in Ref. [20], while in our paper, we have considered the case when the noise strength is smaller than 10^{-3} and larger than 10^{-6}. So, in our paper, we have found that there exists a transition from the chaotic regime to the stochastic regime at a certain critical noise strength. But, in Refs. [19,20], this phenomenon was not found, since they did not consider the case when the noise strength is smaller than 10^{-3}.

In conclusion, we have studied the chaotic noisy transport of electron pairs in a superconducting junction device, a thermal-inertia ratchet. Study shows that when the temperature is low enough, the transport of the electron pairs is mainly chaotic; when the temperature is large enough, it is mainly stochastic. By controlling the temperature and the amplitude of the input ac signal, we can reverse the current of electron pairs. These results help in studies of superconducting junctions and the transport of particles for systems with chaos and noise simultaneously.

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